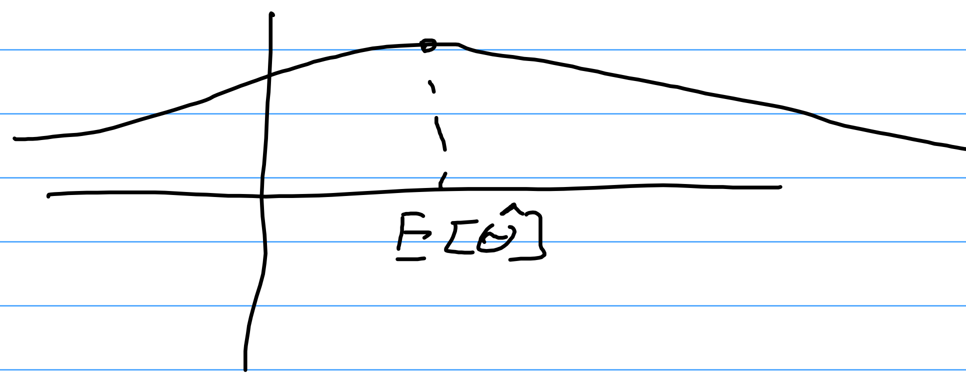
$$MSE(\hat{\theta}) = Var(\hat{\theta}) + (bias(\hat{\theta}))^2$$

$Var(\hat{\theta})$

$$X_1 \quad X_2 \quad X_3$$

$$T(X) = (X_1 + X_3 + X_2)^2 = \hat{\theta}$$



$\hat{\theta} = \theta;$

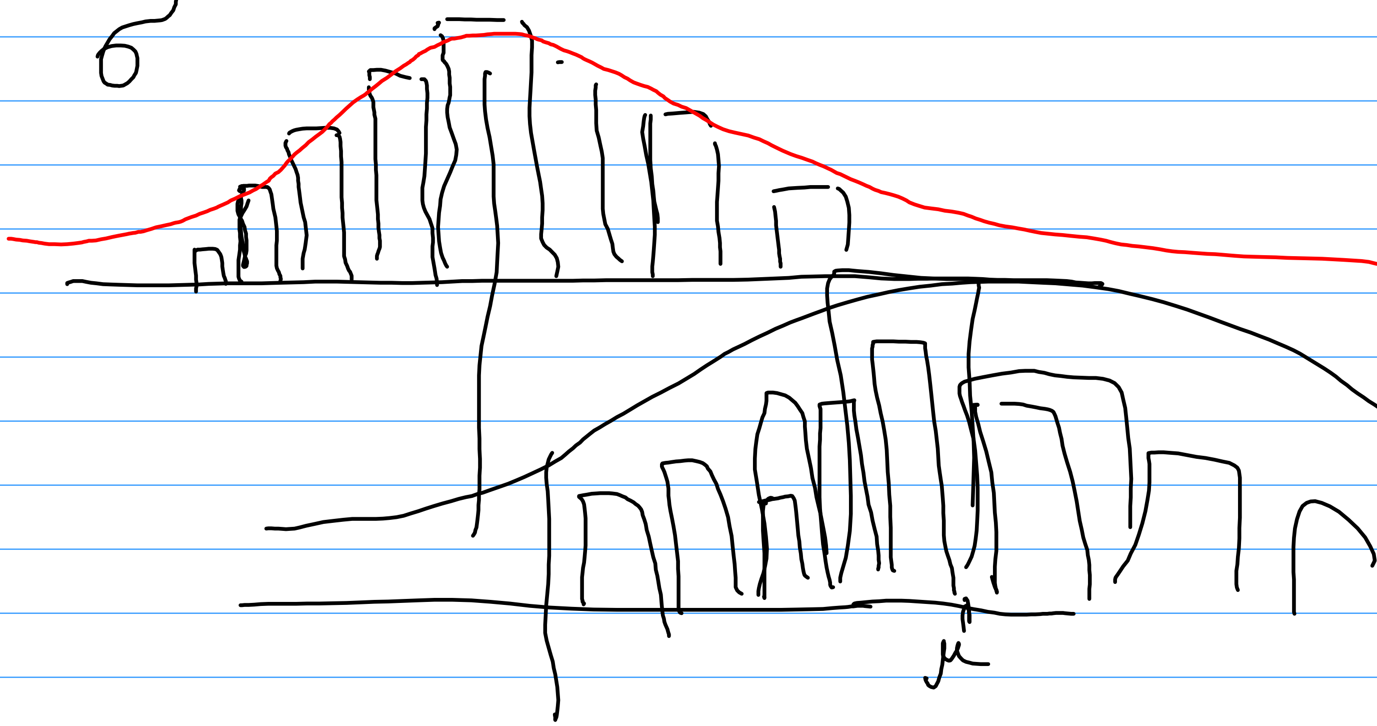
Aim: not precise $\hat{\theta}$

but the region near $\hat{\theta}$ which most likely contains θ

CLT ex.

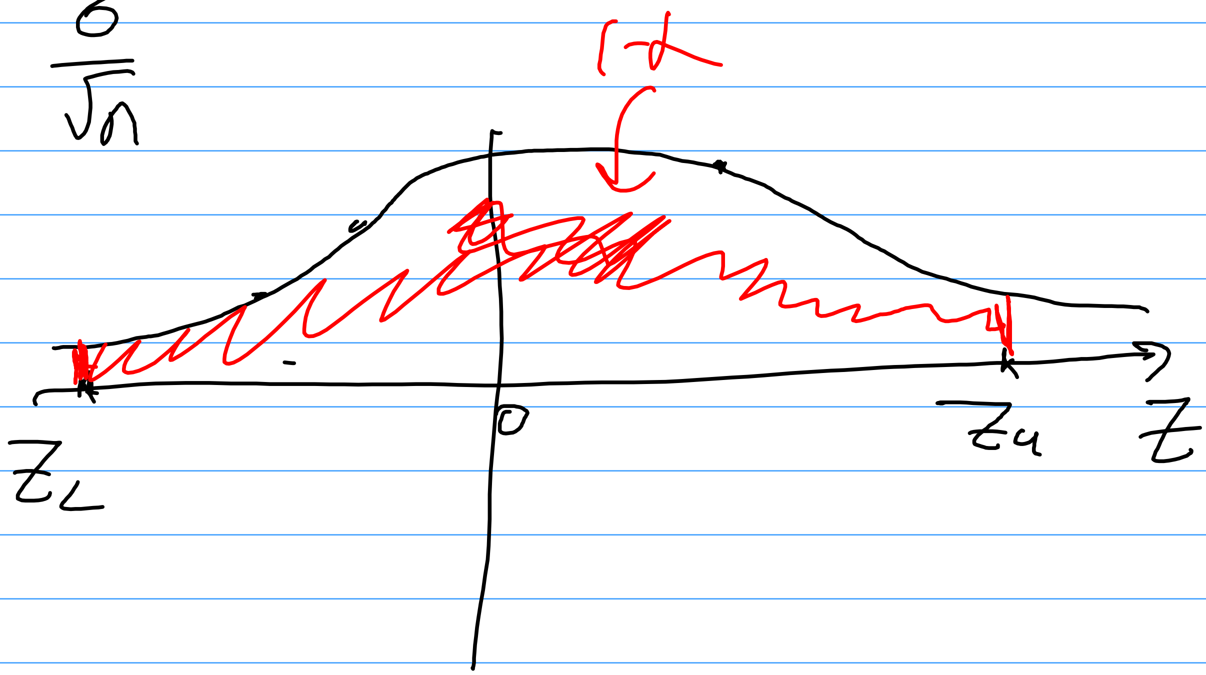
array $= [\bar{X}^{(1)}, \bar{X}^{(2)}, \bar{X}^{(3)}, ----,]$

$$\frac{(\bar{X}^{(i)} - \mu)}{\sigma} \cdot \sqrt{n}$$



$\bar{X}$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad \bar{X} \sim \mu$$



Z_L $\qquad$ Z_u

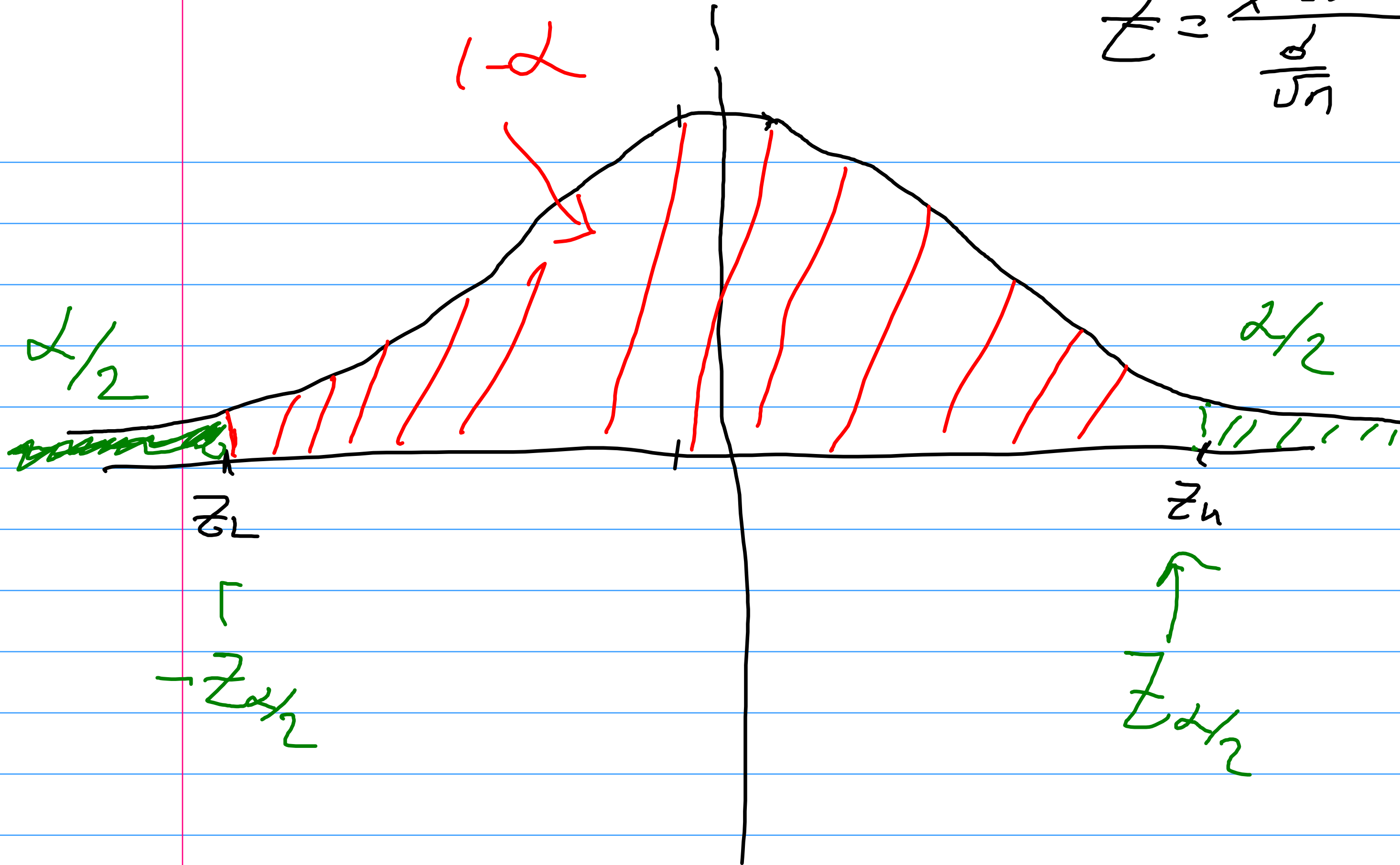
$$P(Z_L < Z < Z_u) = (1-\alpha)$$

$$Z_L < Z$$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > Z_L \Rightarrow \mu < \bar{X} - Z_L \cdot \sim$$

$$P(Z \in (Z_u; +\infty)) + P(Z \in (-\infty; Z_L)) = \alpha$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



$$Z \in \left(-z_{\alpha/2} \,;\, z_{\alpha/2}\right)$$

$(1-\alpha)\cdot 100\%$

$1-\alpha = 0.95$

$\alpha = 0.05$

Proportion

m - number of "positive" results

n - sample size

$\hat{p} = \frac{m}{n}$? p - true proportion in population

sample proportion

CI for proportion

$$p \in \hat{p} \mp E$$

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}}\ ; \ \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

n – sample length.

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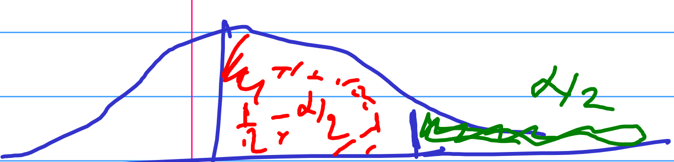
bulbs in factory line

$n = 50$; $m = 5$ $\hat{p} = \frac{1}{10}$

90% CI for the p

$0.9 = 1 - \alpha \rightarrow \alpha = 0.1$

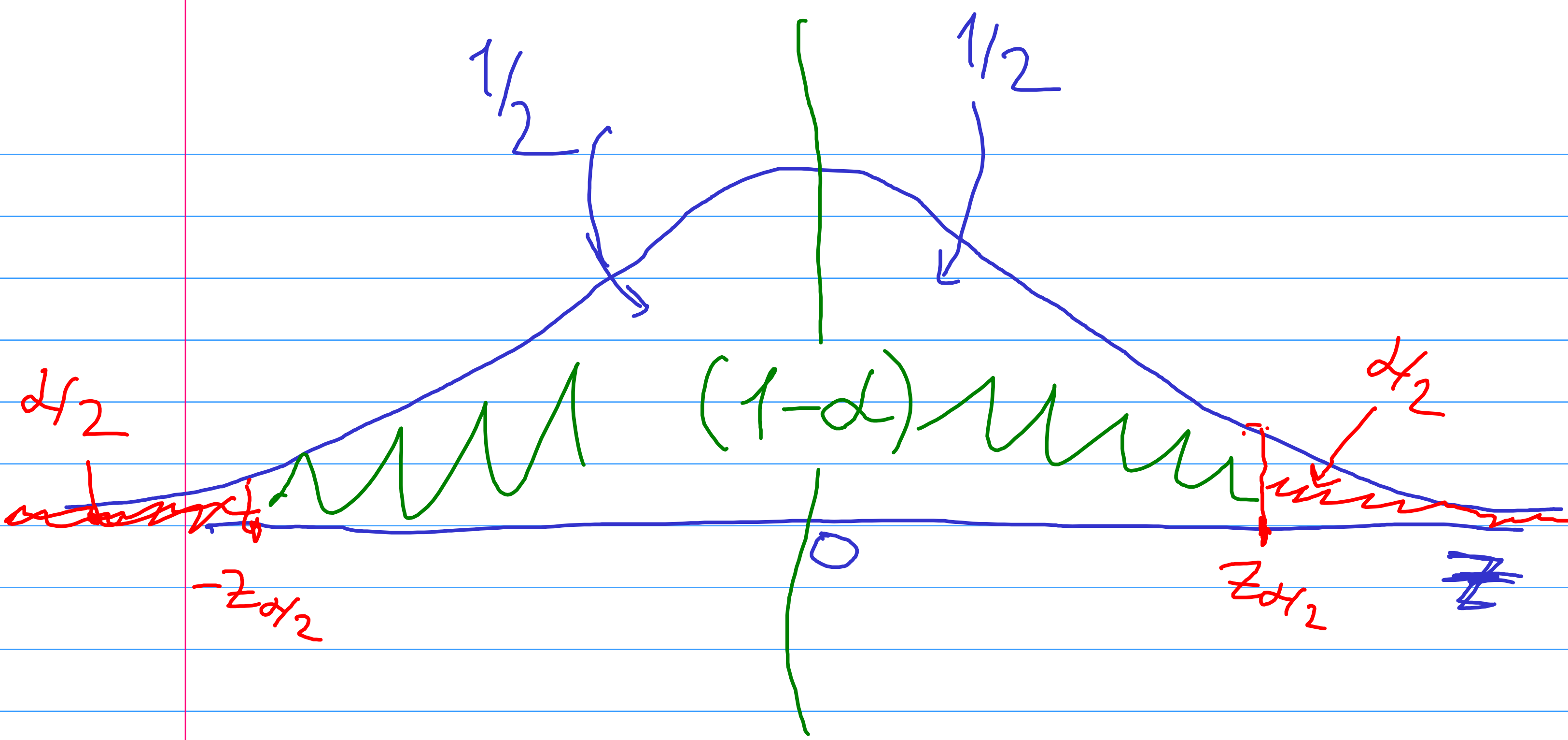
$\alpha/2 = 0.05$; $z_{\alpha/2} = 1.64$



$z_{\alpha/2}$

$$p = 0.1 - 1.64 \cdot \sqrt{\frac{0.9 \cdot 0.1}{50}}\ ; \text{ up to}$$

$$0.1 + 1.64 \sqrt{\frac{0.09}{50}}$$



$z_{\alpha/2}$ – point on z-axis

such that:

$$P\left(Z \in \left(z_{\alpha/2}; +\infty\right)\right) = \alpha/2$$

$$P\left(Z \in \left(-\infty; -z_{\alpha/2}\right)\right) = \alpha/2$$

$$P\left(Z \in \left(-z_{\alpha/2}; z_{\alpha/2}\right)\right) = 1-\alpha$$

$$\underbrace{P\left(Z \in \left(0; z_{\alpha/2}\right)\right)}_{\text{From the table}} + \underbrace{P\left(Z \in \left(z_{\alpha/2}; +\infty\right)\right)}_{\text{need this}} = 1/2$$