Mathematical Statistics

Class 9. Hypothesis Testing: List of scenarios.

MDI, October2022.

Hypothesis about population mean, known population variance

1. Manufacturer of the detergent claims that the contents of boxes sold weight on average at least 16 ounces. The distribution of weights is known to be normal with standard deviation 0.4 ounce. A random sample of sixteen boxes yielded a sample mean weigh of 15.84 ounces. Test at 10% significance level the null hypothesis that the population mean weight is at least 16 ounces.

2. It is reported that the lake water contains 0.5g of salt per 1 liter, with a standard deviation 0.1g. In order to check this statement, 20 samples of water were chosen and the salt in a sample of one liter was 0.57g. Is the report of the salt content correct?

Hypothesis about population proportion

1. A concerned group of citizens wants to show that less than half of the voters supports a new law. Let $p =$ proportion of voters, who supports it.

   - Determine H_0 and H_1.
   - If a random sample of 500 voters gives 228 in support, what does the test conclude? Use $\alpha = 0.05$. Also, evaluate p-value.

2. A random sample of 202 college accounting faculty members was questioned. Of these sample members, 140 felt there was a need for more ethics coverage in accounting courses. Test the null hypothesis that at least 75% of all college accounting faculty members hold this view.

Hypothesis about population mean, unknown population variance

1. A physical model suggests that the mean temperature increase in the water used as a coolant in a compressor chamber should not be more than $5°C$. Temperature increases in the coolant measured on 8 independent runs of the compressing unit revealed the following data:

   $$6.4, 4.3, 5.7, 4.9, 6.5, 5.9, 6.4, 5.1$$

   Does the data contradict to the assertion of the physical model? (Test at $\alpha = 0.05$.) State the assumption you make about the population.

2. A company selling franchises advertises that operators obtain, on average during the, first year, a yield of 10% on their initial investment. A random sample of ten of these franchises produced the following yields for the first year of operation:

   6.1 9.2 11.5 8.6 12.1 3.9 8.4 10.1 9.4 8.9

   Assuming that population yields are normally distributed, test the company's claim.

Test for the difference of population proportions

1. In 1980, the Gallup poll asked Americans whether currently safety regulations made nuclear power plants safe enough. Of the 420 respondents aged 18 to 30, 24% answered "yes". Of the 510 respondents aged 30 to 50, 34% answered "yes".

   (a) Calculate the p-value for H_0 (age makes no difference).

   (b) At level $\alpha = 5\%$, can H_0 be rejected? Is the difference statistically discernible?

2. The Creamery wants to compare adults and children in terms of preference for eating their ice cream out of a cone. They take a representative sample of 500 customers (240 adults and 260 children) and ask if they prefer cones over bowls. They found that 124 adults preferred cones and 90 children preferred cones. Test whether proportions in population of adults and children are close.

Test for difference of population means (σ's unknown, but equal)

1. A trucking firm wishes to choose between two alternative routes for transporting merchandise from one depot to another. One major concern is the travel time. In a study, 5 drivers were randomly selected from a group of 10 and assigned to route A, the other 5 to route B. The following data were obtained.

	Travel time (hours)				
Route A	18	24	30	21	32
Route B	22	29	34	25	35

   Is there significant difference between the mean travel times of the two routes? State the assumptions you have made while performing the test.

2. The table below shows the annual salaries (in \$1000) of randomly selected doctors public medical centers and private hospitals.

State	61	46	68	72	41	59	60	55	40
Private	72	77	54	28	57	24	82		

Test the null hypothesis that mean salary in private hospitals is \$1000 more than in public medical centers. State carefully the assumptions you have made in the test.

Test for difference of population means (paired samples)

1. (UoL 2014) A study was made to determine the amount of fuel economy obtained by using a specific new type of tyre over a standard type. For this reason, 8 cars were fitted with the new type of tyre and the fuel consumption (in km/l) was measured after a test-drive. Afterwards, the same cars with the same drivers were fitted with the standard type tyres and the experiment was repeated to obtain the following fuel consumption measurements.

Car	1	2	3	4	5	6	7	8
Standard type tyres	4.6	5.2	7.4	5.5	5.3	5.2	6.6	6.7
New type tyres	5.1	6.2	7.3	5.4	5.5	5.1	6.1	7.3

   (a) Carry out an appropriate hypothesis test to determine whether the fuel consumption is different between the two types of tyre. State the test hypotheses, and specify your test statistic and its distribution under the null hypothesis. Comment on your findings.

   (b) State any assumptions you made in the latter.

   (c) Give a 95% confidence interval for the difference in means.