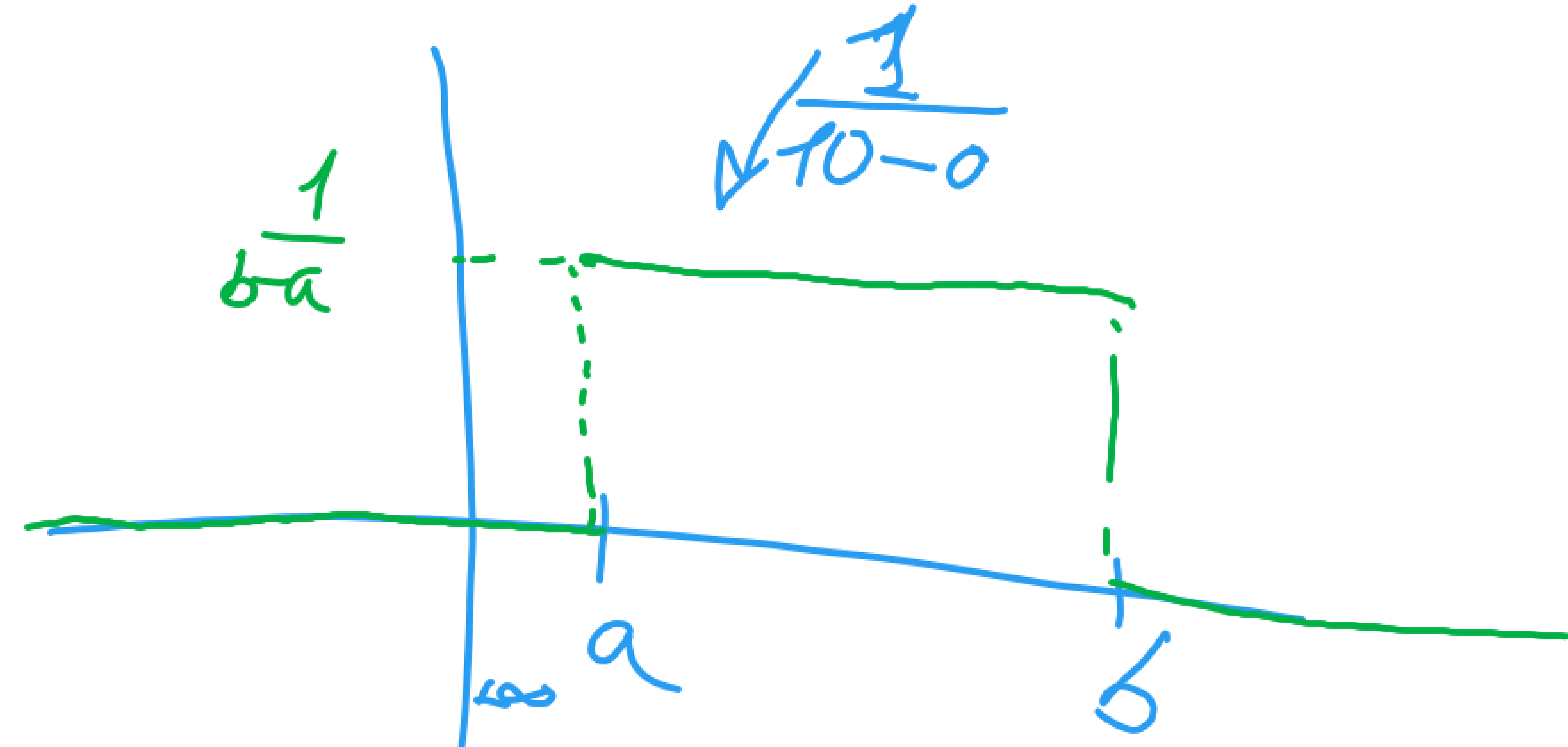
④ $d \sim U(0,10)$

$$f_d = \begin{cases} 0, & d \in (-\infty, a) \\ \frac{1}{b-a}, & d \in (a, b) \\ 0, & d \in (b, +\infty) \end{cases}$$



$$\int_{-\infty}^{+\infty} f_X(x)\,dx = \frac{1}{b-a}\cdot(b-a) = 1$$

$$S(d) = \begin{cases} 10, & 0<d<1 \\ 5, & 1<d<3 \\ 3 & 3<d<5 \\ 0 & 5<d<10 \end{cases}$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x)\,dx$$

$$E[S] = \int_0^1 10\cdot\frac{1}{10}\,dx + \int_1^3 \frac{5}{10}\,dx + \int_3^5 \frac{3}{10}\,dx = 1\cdot x\Big|_0^1 + \frac{1}{2}\cdot 2 + 0.3\cdot 2 = \boxed{2.6}$$

⑥ $T \sim N(60, 10^2)$

$X \sim N$

$P(X + c > a) = \hat{p}$

$P(X > a - c) = \hat{p}$

t - time leaving home

$P(t + T > 9) = 0.2$

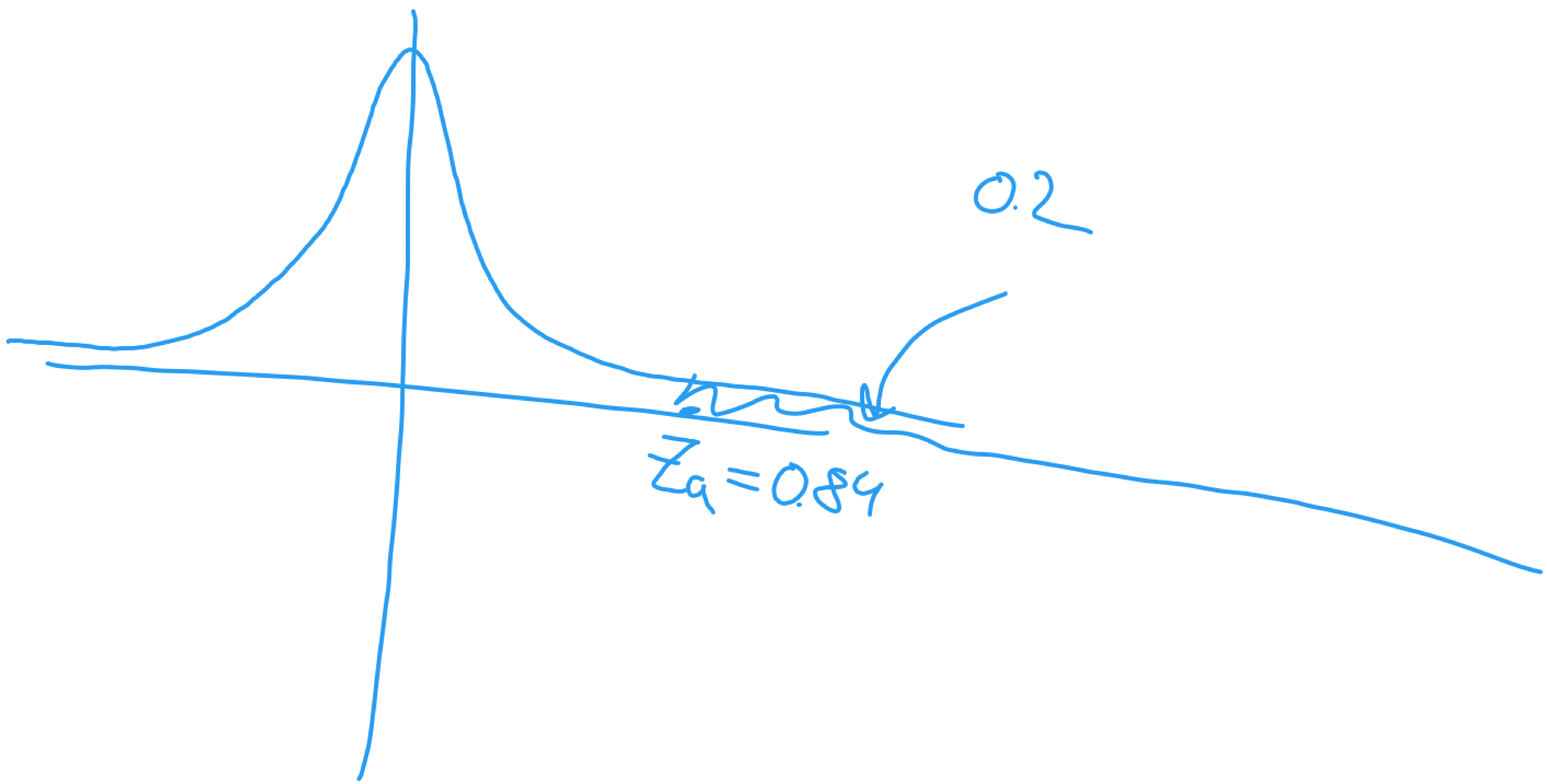
$P(T > \underbrace{9 - t}_{a}) = 0.2$

$P(T > a) = 0.2$

$P(Z > \underbrace{\frac{a-60}{10}}_{Z_a}) = 0.2$

$\frac{a-60}{10} = 0.84 \rightarrow a = 68.4$

$t = 9 \text{ a.m.} - 68.4 \text{ min} = 7.51\ldots \text{ A.M}$


0.2
$z_a = 0.84$

joint P.M.F.

$$P(X=x_i,\ Y=y_j)$$

	$Y=y_1$	...	y_m
$X=x_1$	p_{11}	...	p_{1m}
$\vdots$			
x_n	p_{n1}	...	p_{nm}

$P(X=x_i)$ – marginal p.m.f.

$P(Y=y_j)$

$$P(X=x_i)=\sum_{j=1}^{m} P(X=x_i;\ Y=y_j)$$

Dependence

$$P(X=x_i,\ Y=y_j)=P(X=x_i)\cdot P(Y=y_j)\ \forall i,j$$

$$\mathrm{Cov}(X,Y) = E[X \cdot Y] - E[X] \cdot E[Y]$$

$$E[g(X,Y)] = \sum_i \sum_j g(x_i, y_j) \cdot P(X = x_i, Y = y_j)$$

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X) \cdot \mathrm{Var}(Y)}}$$

$$\mathrm{corr} = 1 \rightarrow Y = aX + b$$

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	$Y=$ -1	0	1	
X -1	0.2	0	0.2	0.4
0	0	0.2	0	0.2
1	0.2	0	0.2	0.4
	0.4	0.2	0.4	

$P(X=-1, Y=-1) = 0.2$

$\neq$

$P(X=-1) \cdot P(Y=-1) = 0.16$

Dependent

$E[X \cdot Y] = 0.2 - 0.2 + 0 + 0.2 - 0.2 = 0$

$E[X] = 0.4 - 0.4 = 0$

$Cov(X,Y) = 0$!!!

⑨ $X+Y$

$$\mathrm{Var}(X\pm Y) = E\left[\left(X\pm Y - E[X\pm Y]\right)^2\right] =$$

$$= E\left[\left((X-E[X]) \pm (Y-E[Y])\right)^2\right] = E\left[(X-E(X))^2 + (Y-E(Y))^2 \pm 2(X-E(X))\cdot(Y-E(Y))\right] =$$

$$= \mathrm{Var}(X) + \mathrm{Var}(Y) \pm 2\underbrace{E[(X-E(X))\cdot(Y-E(Y))]}_{\mathrm{cov}(X,Y)}$$

⑦ $X \sim N(1, 3)$

$Y \sim N(0, 4)$

$A = X - Y$

$A \sim N(1, 7)$

$E(X-Y) = E(X) - E(Y) = 1 - 0 = 1$

$Var(X-Y) = Var(X) + Var(Y) = 7$

$P(X > Y)$

$P(\underbrace{X - Y}_{A} > 0)$

$P(A > 0)$

$P\left(Z > \frac{-1}{\sqrt{7}}\right) = 0.648$

-0.377