12

$X \backslash Y$	0	1	2	
0	0.1	0.08	0.02	0.2
1	0.06	0.09	0.15	0.3
2	0.04	0.13	0.33	0.5
	0.2	0.3	0.5	

$$P(X=x_i \mid Y=y_j) = \frac{P(X=x_i; Y=y_j)}{P(Y=y_j)}$$

$$P(X=x_i; Y=y_j) = P(X=x_i \mid Y=y_j)\cdot P(Y=y_j)$$

$$P(X=0; Y=0) = P(Y=0 \mid X=0)\cdot P(X=0)$$

$$E[XY] = \sum_{i=1}^{3}\sum_{j=1}^{3} x_i \cdot y_j \cdot P(X=x_i, Y=y_j)$$

7

$Var(X+Y) = Var(X) + Var(Y)$

$E[X-Y] = E[X] - E[Y]$

$X \sim N(1, 3)$

$Y \sim N(0, 4)$

$K \sim N(1, 7)$

$P(X > Y)$

$P(\underbrace{X - Y}_{K} > 0)$

$\downarrow$

$P\left(Z > \underbrace{\frac{0-1}{\sqrt{7}}}_{-0.378}\right) \approx 0.648$

$Var(X-Y) = Var(X) + Var(Y)$

(13) X, Y have joint normal $\longrightarrow$ $aX + bY$, $a, b \in \mathbb{R}$ $\longrightarrow$ $N(\ldots, \ldots)$

$corr(X,Y) - ?$

$Var(X+Y) = Var(X) + Var(Y) + 2 \cdot cov(X,Y)$

$P(X+Y > 1.96) = 0.05$

$K = X+Y$; $K \sim N(0, \sigma^2)$

$P(K > 1.96) = 0.05$

$P\left(Z > \underbrace{\frac{1.96-0}{\sigma}}_{Z_K}\right) = 0.05$

$Z_K = 1.64$

$\sigma = \frac{1.96}{1.64} \approx 1.2$

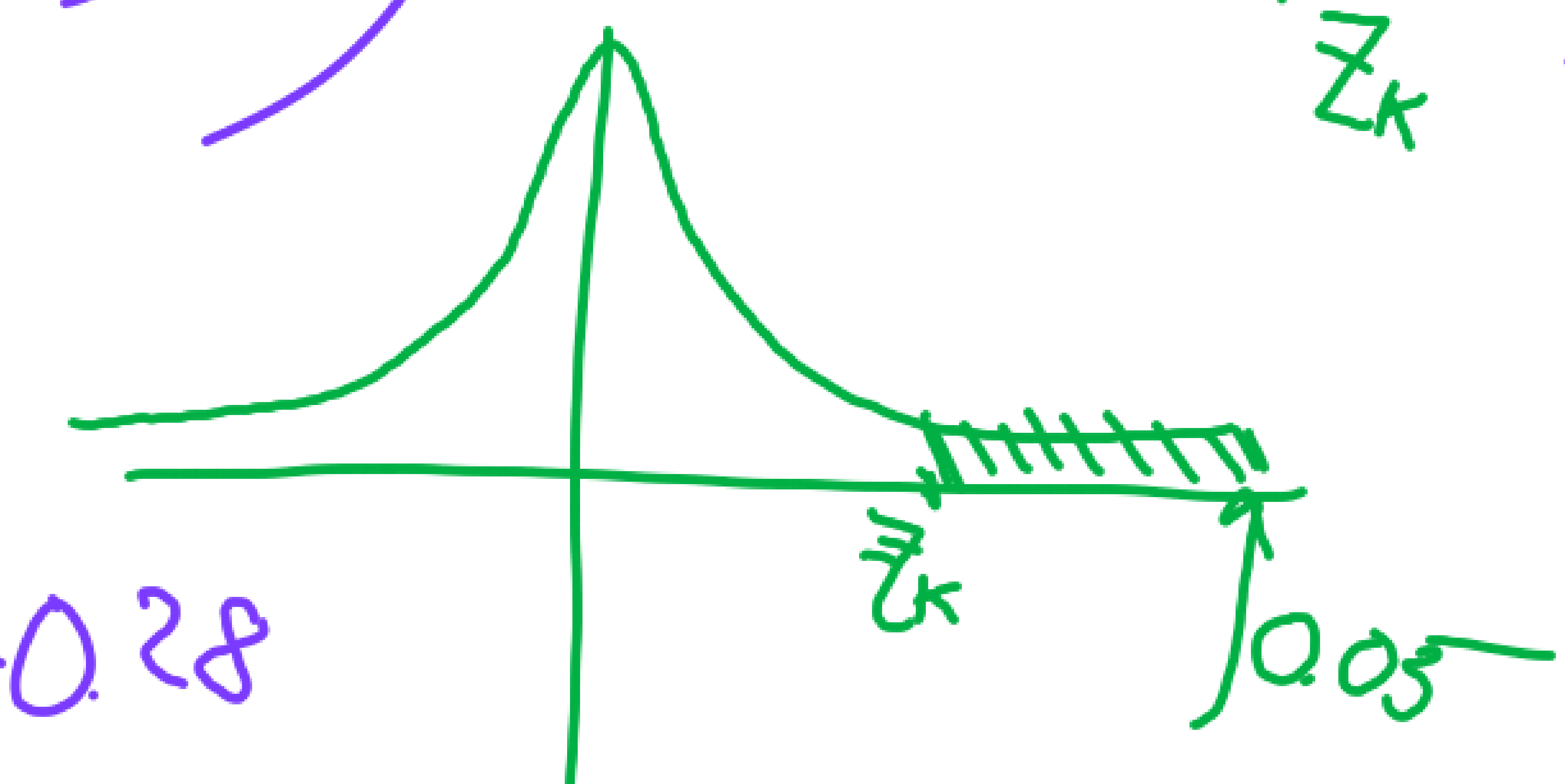
$E[X+Y] = E(X) + E(Y)$

$N(0,1)$

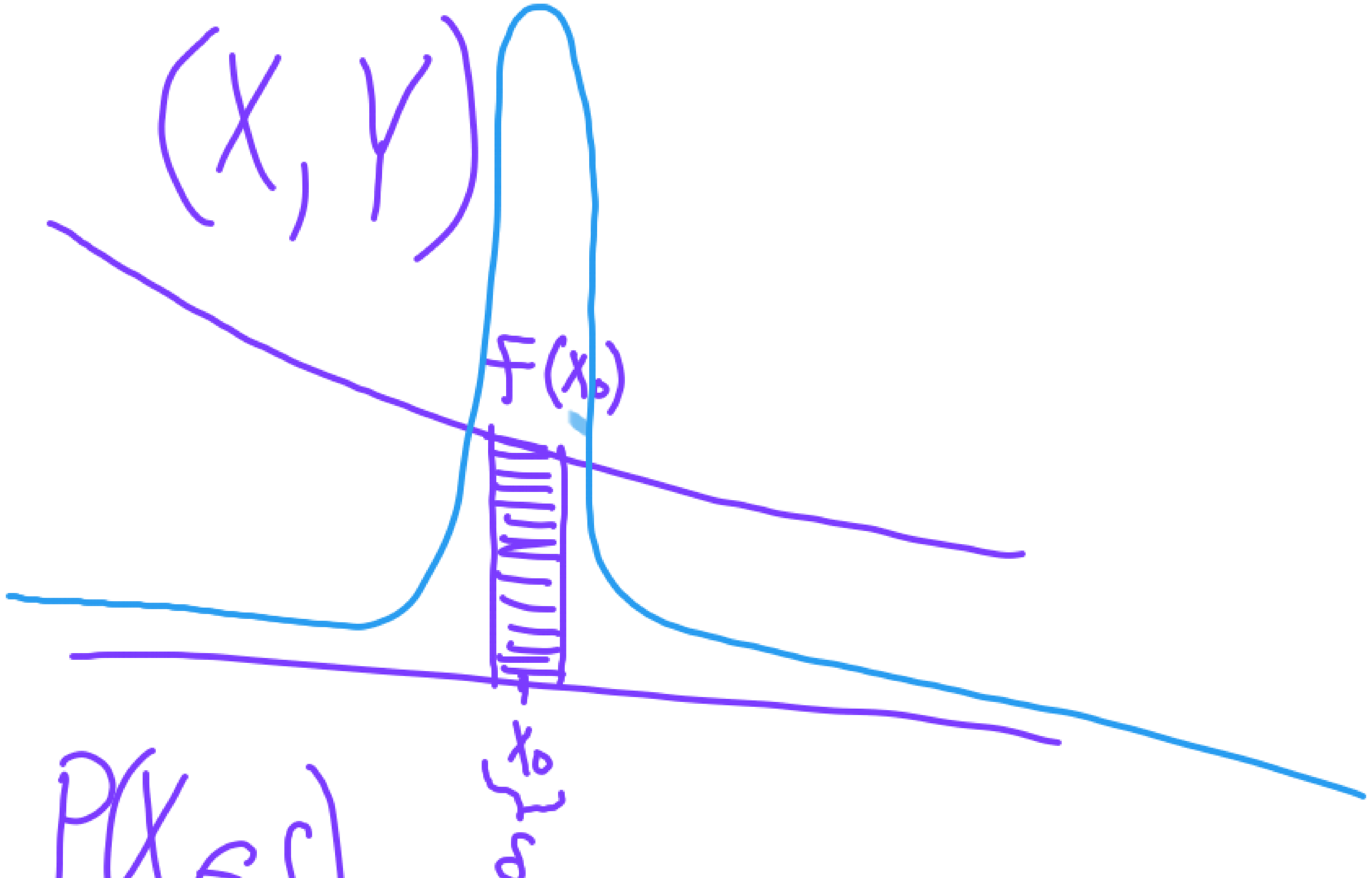
$1.44 = 1 + 1 + 2\,cov(X,Y)$

$cov(X,Y) = -0.28$

$Cor(X,Y) = \frac{-0.28}{1 \cdot 1} = -0.28$

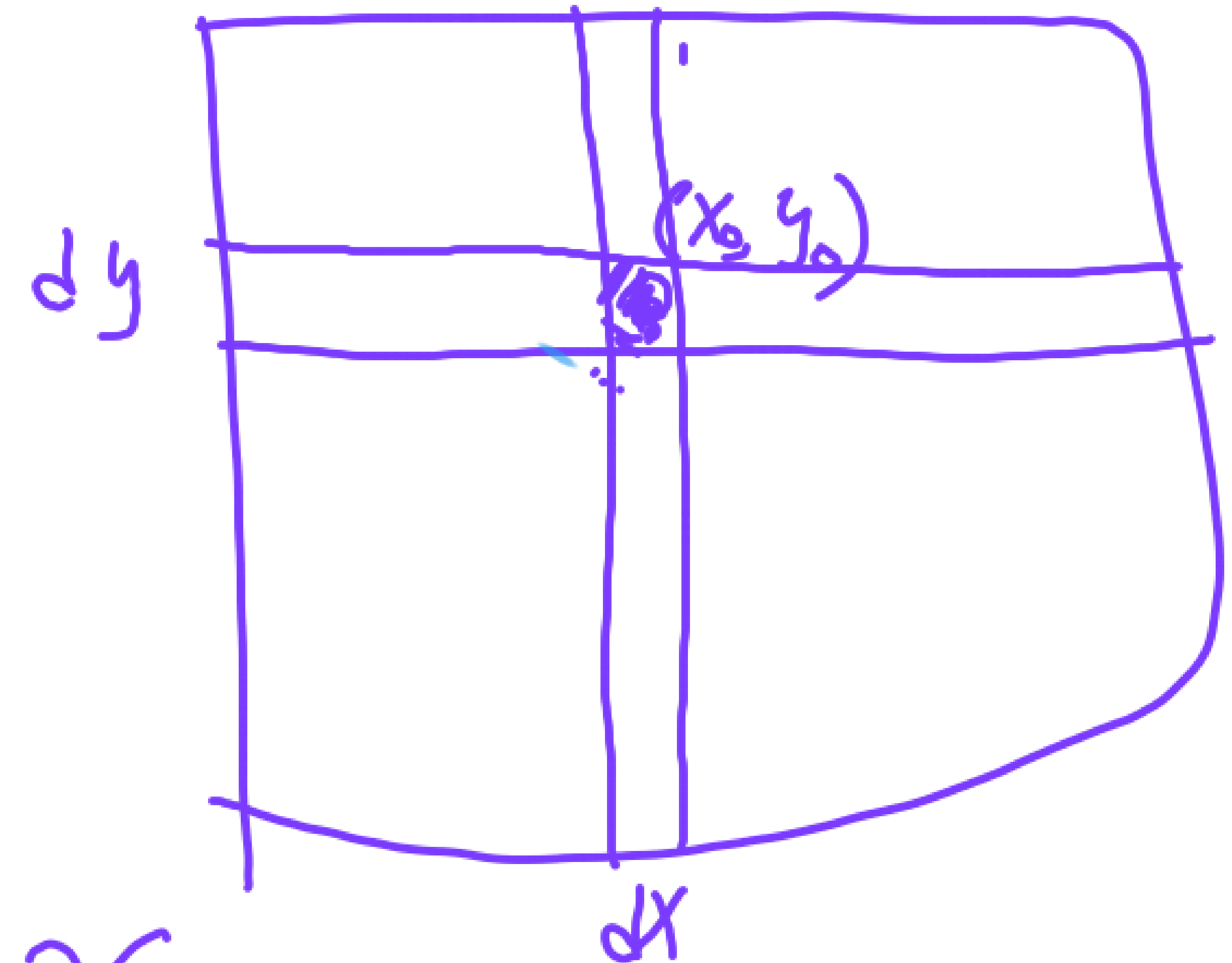


(X, Y)



$$P(X \in \delta) = \delta \cdot f(x_0)$$

$$P(X \in [a,b]) = \int_a^b f(x)\,dx$$



$$P(X \in dx, Y \in dy) = dx \cdot dy \cdot f(x_0, y_0)$$

$$P(X \in [a,b]; Y \in [c,d]) = \int_a^b \int_c^d f(x,y)\,dx\,dy$$

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

$$P(X=x_i) = \sum_{j=1}^{m} P(X=x_i ; Y=y_j)$$

$$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\, dx$$

$\mathcal{N}(\mu, \sigma^2)$

$Bin(n, p)$

$Exp(\lambda)$

Θ

$X_i \sim F(x|\Theta)$

$(X_1, X_2, X_3, \dots, X_n)$

I.I.D.

Independent

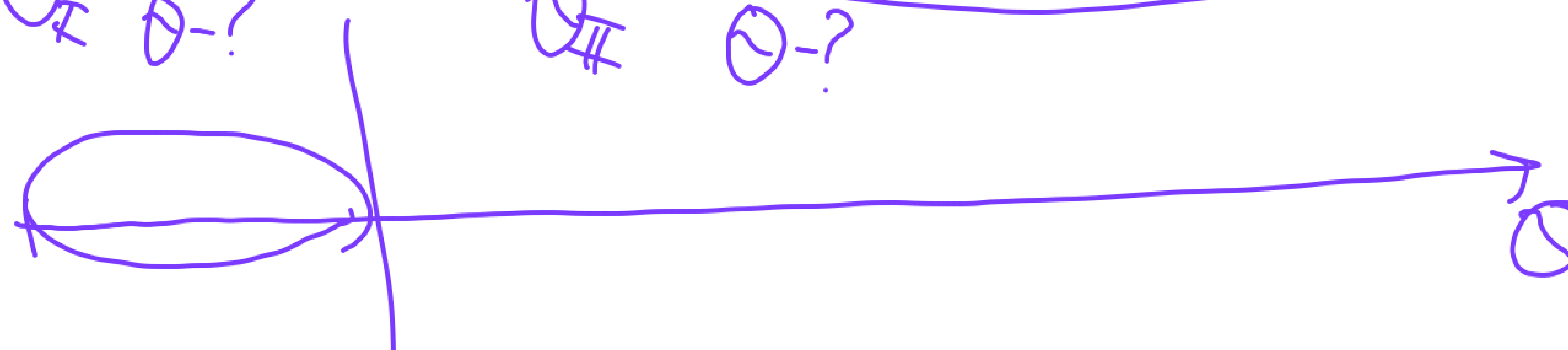
Ident.

Distributed

$\hat{\Theta} = F(X_1, \dots, X_n) \to \Theta$

Θ_I Θ-?

Θ_{II} Θ-?

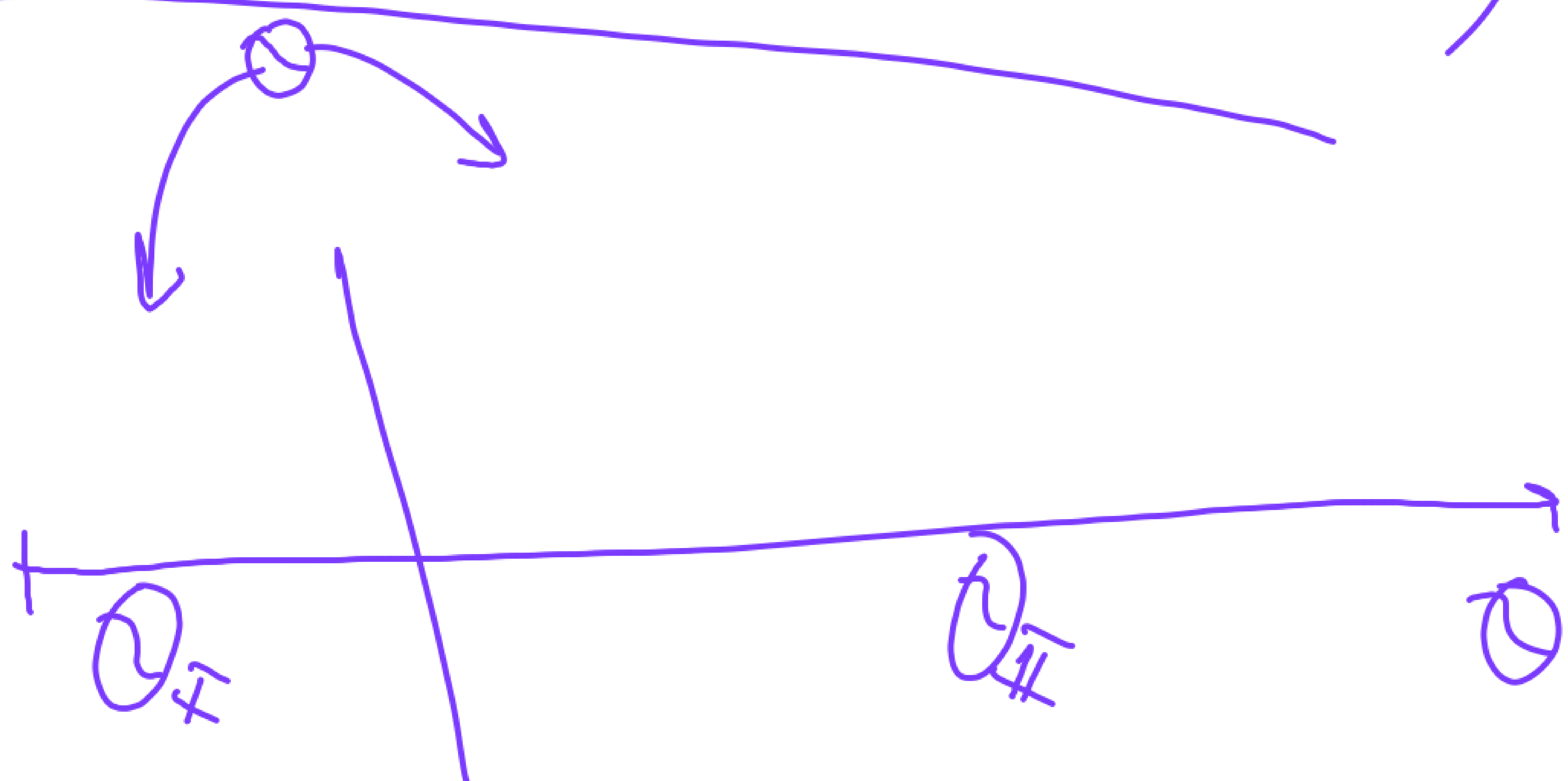


$\boxed{X \sim F(x|\theta)}$; $N(\mu, \sigma^2)$

$Bin(n, p)$

$Exp(\lambda)$

θ - ?



$\boxed{\text{I.I.D.}}$

Indep. Ident. Distributed

$(X_1, X_2, X_3, \ldots, X_n)$

$X_i \sim F(X \mid \theta)$ unknown

$T(X_1 \ldots X_n)$ — Statistic

$\hat{\theta}$ — point estimator

$\hat{\theta} = T(X_1, \ldots, X_n)$

$N(\mu, \sigma^2)$

$N(0, 1)$

$\underline{\text{Bias}} = E[\hat{\theta}] - \theta$; $\boxed{E[\hat{\theta}] = \theta}$

$\underline{X_1, X_2, X_3}$ $\boxed{T = X_1 + X_2 + X_3}$ $E[(\theta - \hat{\theta})^2] = MSE$

$X_1^{(2)}, X_2^{(2)}, X_3^{(2)}$; $\bar{T} =$