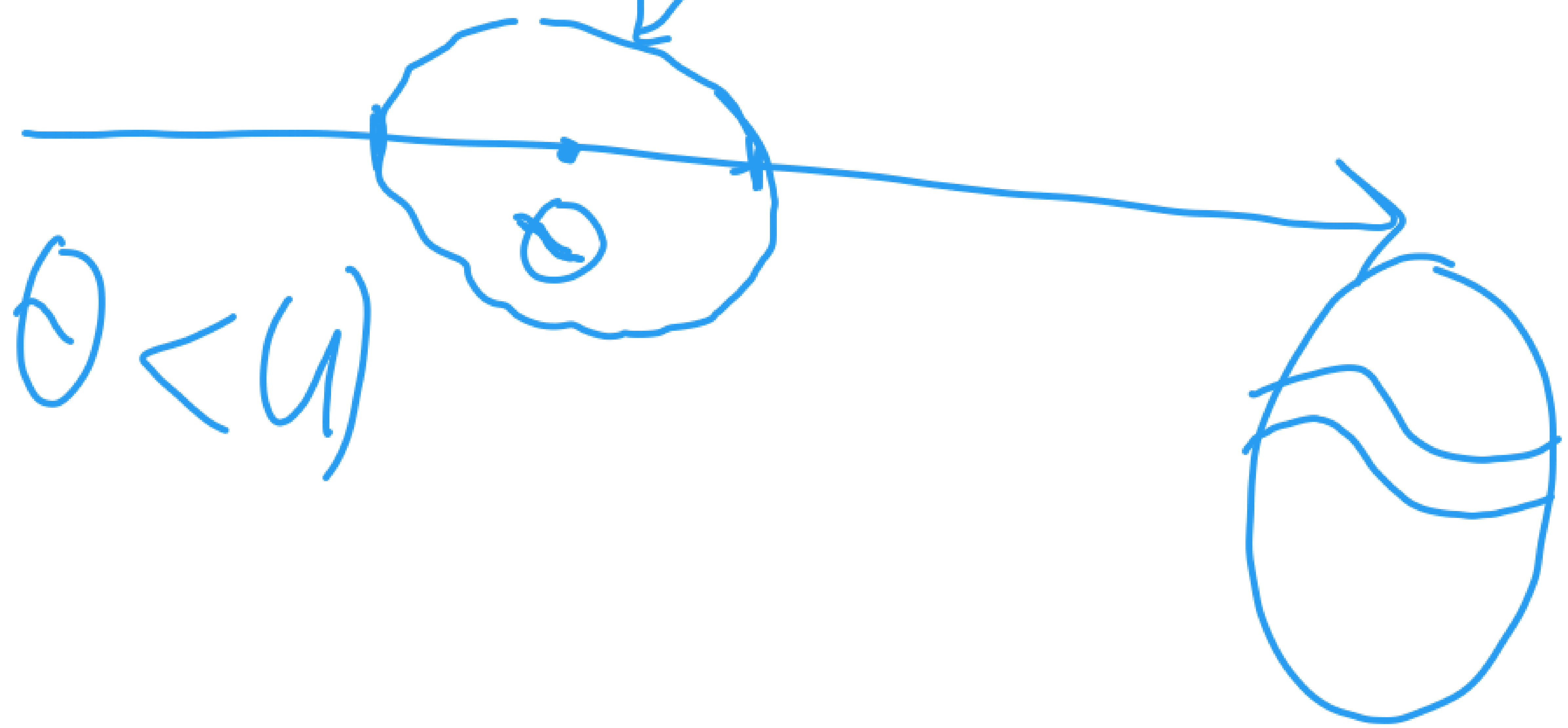
$$P(\hat{\theta} = \theta) = 0 \qquad \hat{\theta} = T(X_1, \dots, X_n)$$

$(1-\alpha)\cdot 100\%$

$\alpha = 0.05$

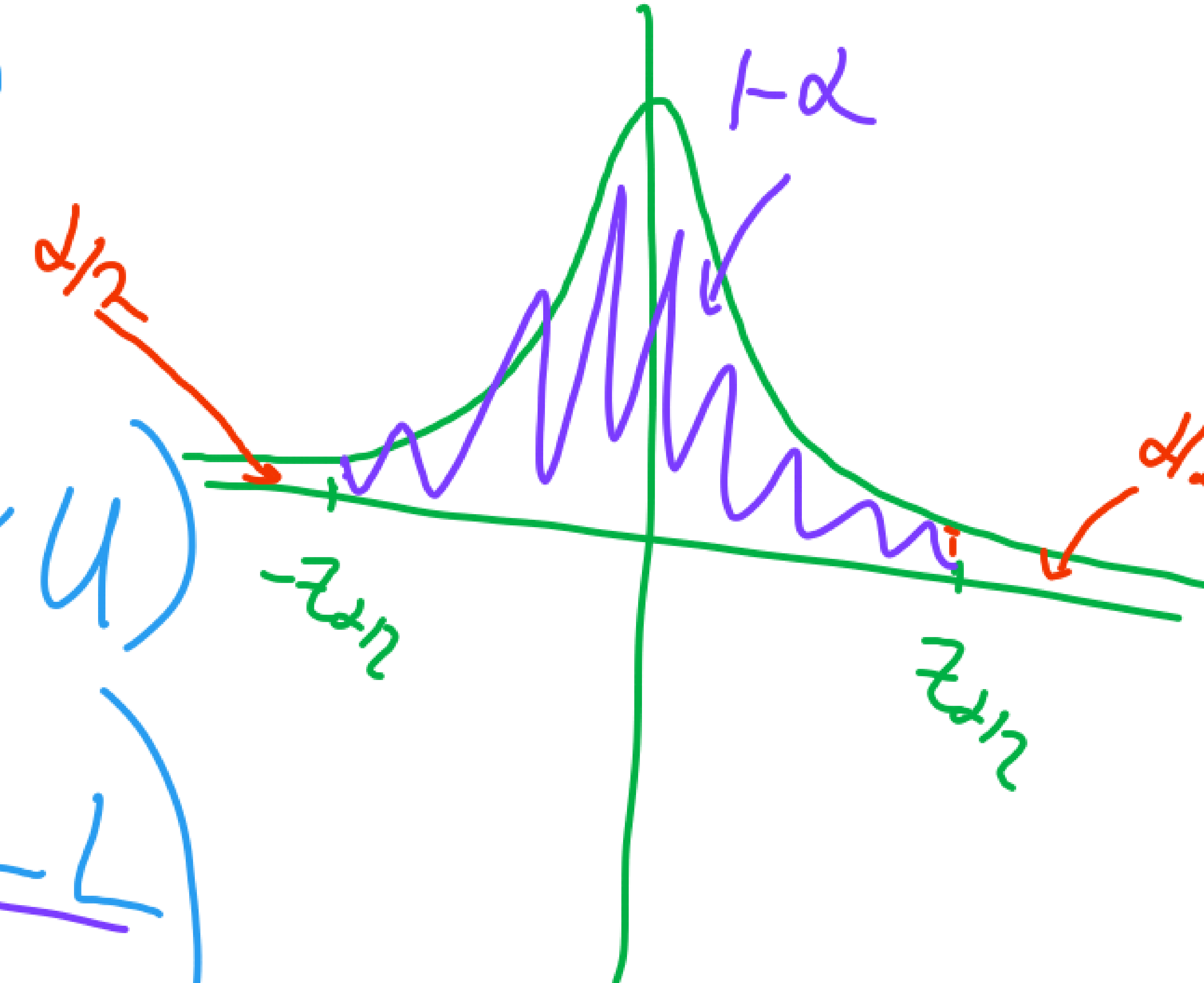
$$\boxed{1-\alpha} = P(L < \theta < U)$$



Confidence interval for the population mean

$\boxed{\mathrm{I}}$ $\mu$, $\sigma^2$ – known $X_1, \dots, X_n$

$$1-\alpha = P(L < \mu < U)$$




$$1-\alpha = P\left(\underbrace{\frac{\bar{X}-U}{\frac{\sigma}{\sqrt{n}}}}_{-z_{\alpha/2}} < \underbrace{\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}}_{Z} < \underbrace{\frac{\bar{X}-L}{\frac{\sigma}{\sqrt{n}}}}_{z_{\alpha/2}}\right)$$

$$L = \bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$U = \bar{X} + \text{—//—}$$

② $X_1 \dots X_{36}$ $\quad (\overset{L}{3.5}, \overset{U}{3.7})$

$$\bar{X} = 3.6$$

$$\sigma = 0.72$$

$$\alpha \longrightarrow L, U$$

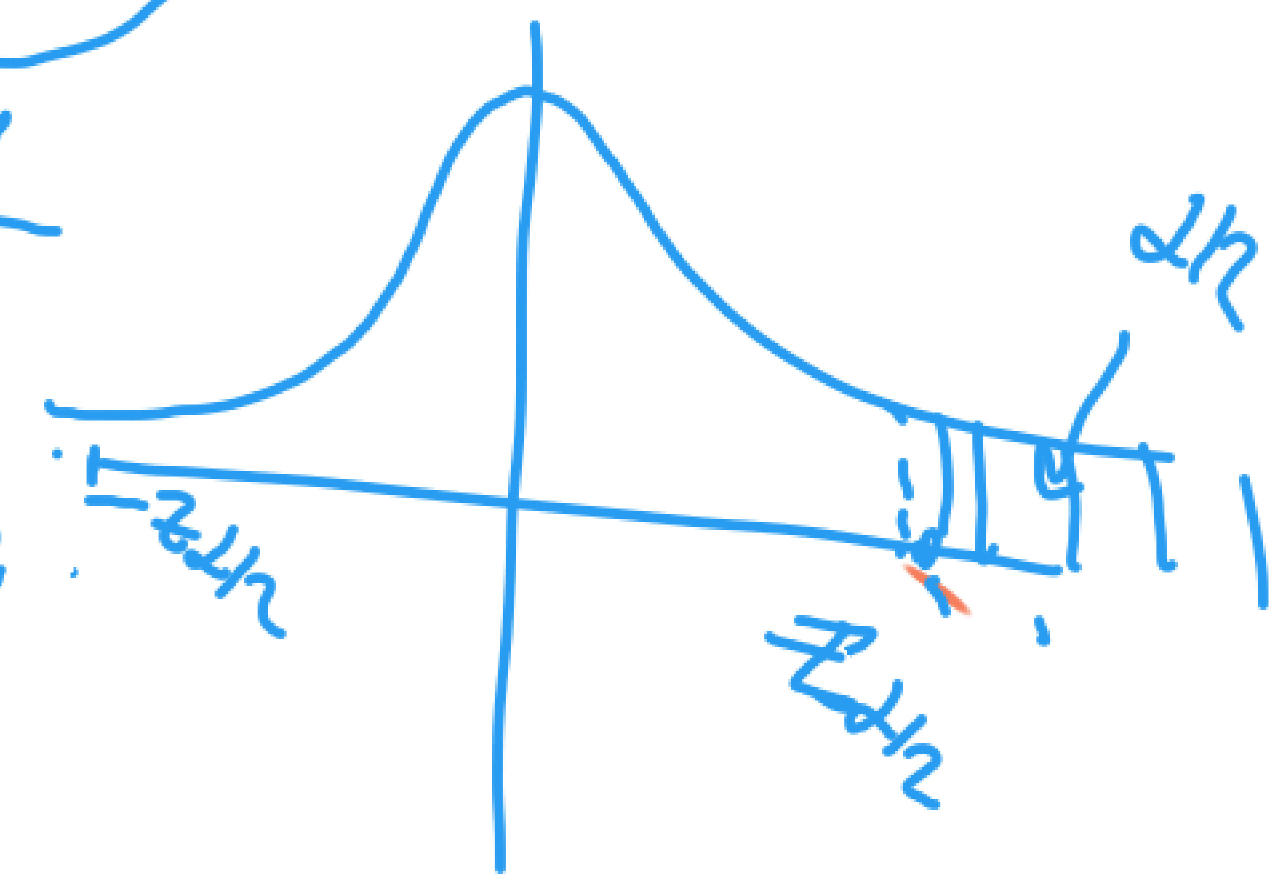
$$\alpha \longleftarrow L, U$$

$$(1-\alpha)\cdot 100\% = 59.4\%$$

$$\underset{3.7}{U} = \underset{3.6}{\bar{X}} + \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{0.1}$$

$$\underset{3.5}{L} = \underset{3.6}{\bar{X}} - \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{0.1}$$

$$z_{\alpha/2} = \frac{0.1 \cdot 6}{0.72} = 0.83$$



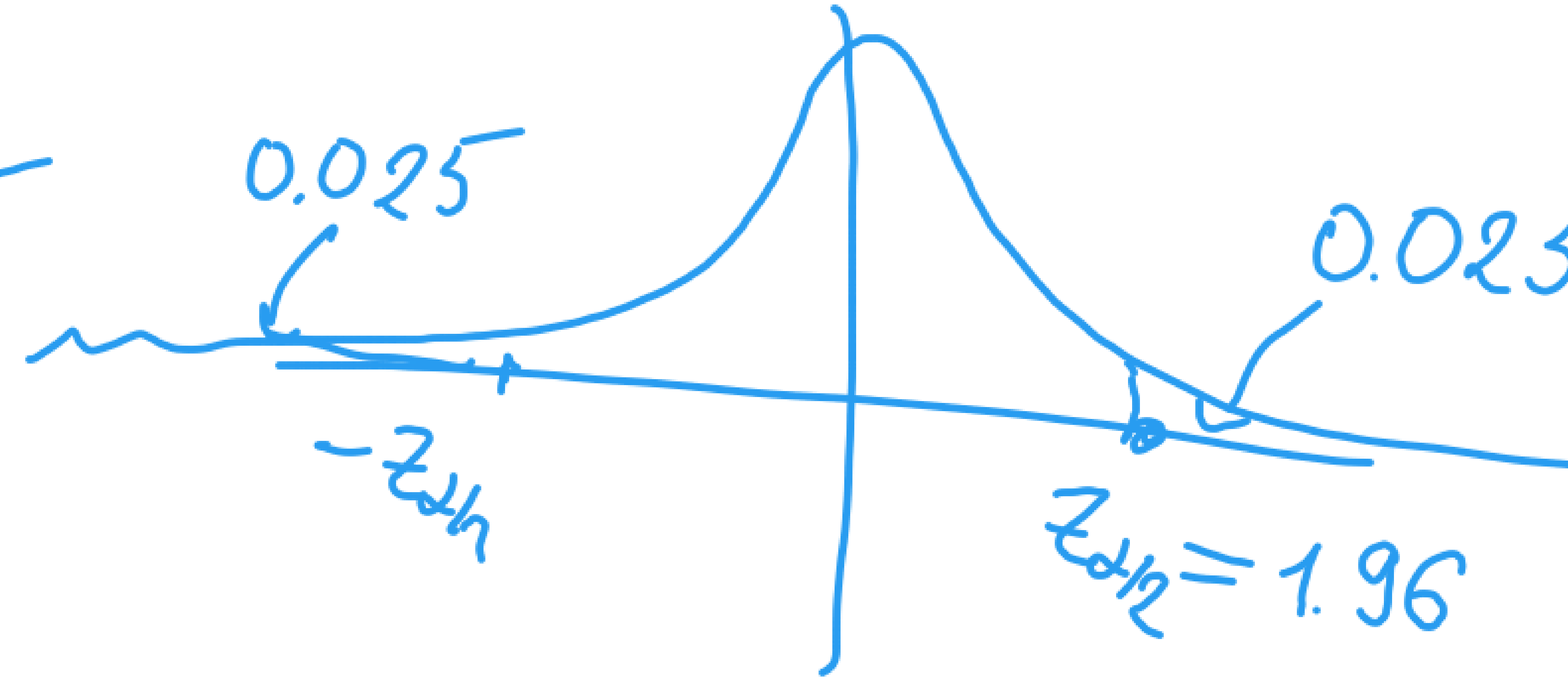

$$P(Z < 0.83) = 0.797$$

$$\alpha/2 = 1 - 0.797 = 0.203$$

$$\alpha = 0.406$$

3.1. $X_1 \dots X_{25}$

$X_i \sim N(\mu, (0.45)^2)$




$\bar{X} = 2.9$

95%

$\alpha = 0.05$

$$\mu \in \left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}};\ \bar{X} + \text{—//—}\right)$$

$$\mu \in \left(2.9 - 1.96 \cdot \frac{0.45}{5};\ 2.9 + \text{—//—}\right)$$

3.2.

$(2.81; 2.99)$ $2.99$ $2.9$

$\alpha - ?$

$$u = \bar{x} + \underbrace{z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{0.09}$$

$$z_{\alpha/2} = \frac{0.09\cdot 5}{0.45} = 1.$$

$$(1-\alpha)\cdot 100\% = 68\%$$


$$P(-\infty < z < 1) = 0.841$$

$$\alpha/2 = 1 - 0.841 = 0.159$$

$$\alpha \approx 0.32$$

# Confidence Interval for population proportion

$\boxed{\text{II}}$ $X_1, \ldots, X_n$, $\underline{n > 30}$

$$\hat{P} = \frac{k}{n} \qquad \hat{P} \sim N\left(P, \frac{P(1-P)}{n}\right)$$

$$1-\alpha = P(L < p < U)$$

$$1-\alpha = P\left(\underbrace{\frac{\hat{p} - U}{\sqrt{\frac{p(1-p)}{n}}}}_{-z_{\alpha/2}} < \underbrace{\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}}_{Z} < \underbrace{\frac{\hat{p} - L}{\sqrt{\frac{p(1-p)}{n}}}}_{z_{\alpha/2}}\right)$$

$$U = \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$L = \hat{p} - \text{---} // \text{---}$$

4

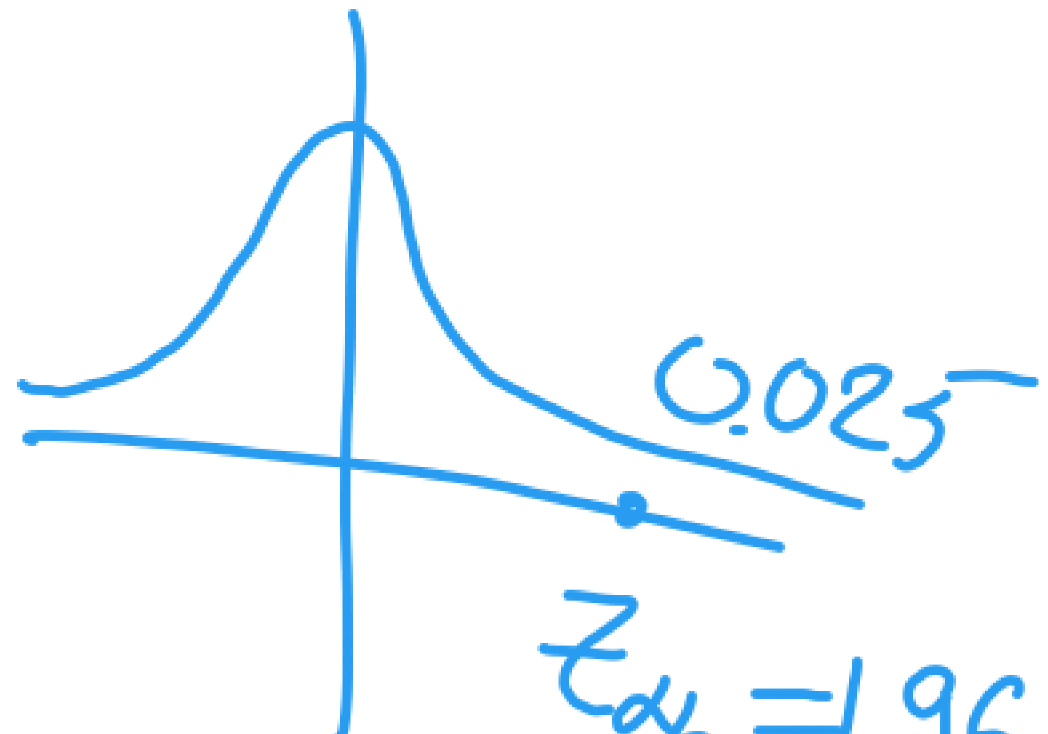
1963 $\frac{160}{200}$

95% CI

$p,\ (1-p)=q$

$$P \in \left(\hat{p} - z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}\ ;\ \hat{p} + \text{—//—}\right)$$

1968 $\frac{70}{200}$




$z_{\alpha/2} = 1.96$

$$P \in \left(0.8 - 1.96\sqrt{\frac{0.8\cdot 0.2}{200}}\ ;\ 0.8 + \text{—//—}\right)$$

$$P \in \left(0.35 - 1.96\sqrt{\frac{0.35\cdot 0.65}{200}}\ ;\ 0.35 + \text{—//—}\right)$$

(Version from the class in another group)

1963

$$\boxed{\frac{160}{200}} = \hat{p}$$

95% CI

$\alpha = 0.05$

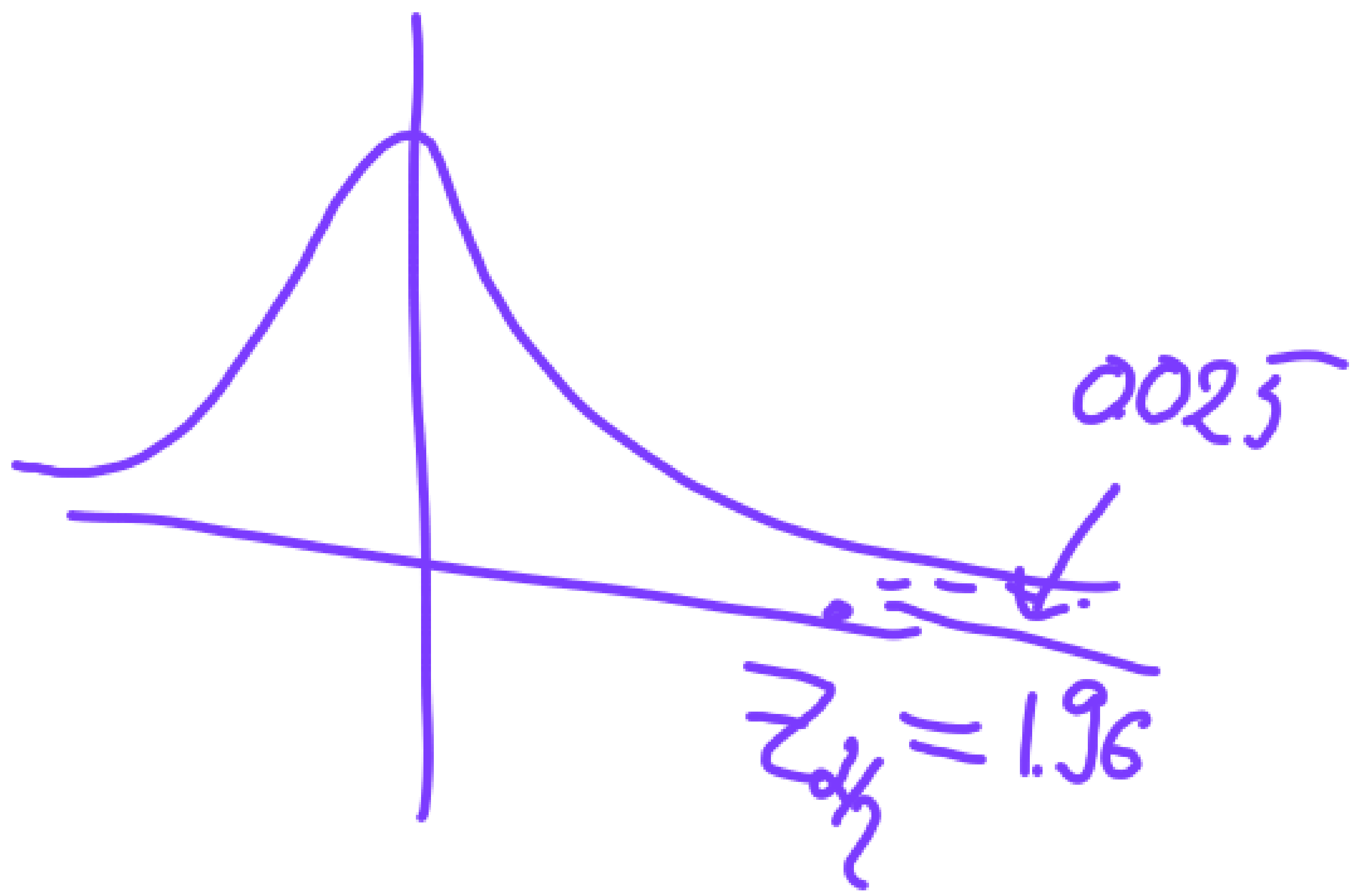
$(1-0.05)\cdot 100\% = 95\%$

$\hat{q} = (1-\hat{p})$

$$p \in \left(\hat{p} - z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}\,;\ \hat{p} + z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}\right)$$

1968

$$\frac{70}{200}$$




1963:

$$p \in \left(0.8 - 1.96\cdot\sqrt{\frac{0.8\cdot 0.2}{200}}\,;\ 0.8 + 1.96\sqrt{\frac{0.8\cdot 0.2}{200}}\right)$$

1968

$$p\left(0.35 - 1.96\sqrt{\frac{0.35\cdot 0.65}{200}}\,;\ 0.35 + \text{—//—}\right)$$

III

$X_1 \, - \, - \, - \, X_n \sim F(\mu_1, \sigma_1^2)$    $\bar{X}$

$\sigma_1, \sigma_2$ - known

$E[X-Y] = E[X] - E(Y)$

$Var(X-Y) = Var(X) + var(Y)$

$Y_1 \, - \, - \, - \, Y_m \sim F(\mu_2, \sigma_2^2)$    $\bar{Y}$

$\bar{X} \sim N\left(\mu_1, \frac{\sigma_1^2}{n}\right)$

$\bar{Y} \sim N\left(\mu_2, \frac{\sigma_2^2}{m}\right)$

$$1-\alpha = P(L < \overbrace{\mu_1 - \mu_2}^{\theta} < U)$$

$$\overbrace{\bar{X} - \bar{Y}}^{\hat{\theta}} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right)$$

$$\underbrace{\frac{\hat{\theta} - U}{-//-}}_{-z_{\alpha/2}} < \underbrace{\frac{\hat{\theta} - \theta}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}}_{Z} < \underbrace{\frac{\hat{\theta} - L}{-//-}}_{z_{\alpha/2}}$$

$$U = \bar{X} - \bar{Y} + z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}$$

$$L = \bar{X} - \bar{Y} \quad \text{———} \quad -//-$$

## IV Diff. of popul. proportions

$X_1 \ldots X_n \qquad \hat{p}_1 = \frac{k}{n}$

$Y_1 \ldots Y_m \qquad \hat{p}_2 = \frac{r}{m}$

$$1-\alpha = P\left(L < \overbrace{p_1 - p_2}^{\theta} < U\right)$$

$$\hat{p}_1 \sim N\left(p_1, \frac{p_1 q_1}{n}\right)$$

$$\hat{p}_2 \sim N\left(p_2, \frac{p_2 q_2}{m}\right)$$

$$\underbrace{\hat{p}_1 - \hat{p}_2}_{\hat{\theta}} \sim N\left(p_1 - p_2, \frac{p_1 q_1}{n} + \frac{p_2 q_2}{m}\right)$$

$$L = \hat{p}_1 - \hat{p}_2 - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n} + \frac{\hat{p}_2 \hat{q}_2}{m}}$$

$$U = \hat{p}_1 - \hat{p}_2 + \text{— // —}$$

(Version from the class in another group)

Diff between popul. proportions

$E(X-Y) = E(X) - E(Y)$

$Var(X-Y) = Var(X) + Var(Y)$

$X_1 \dots X_n, \; Y_1 \dots Y_m$

$\hat{p}_1 \qquad \hat{p}_2$

$p_1 - p_2$

$\hat{p}_1 \sim N\left(p_1, \frac{p_1(1-p_1)}{n}\right)$

$\hat{p}_2 \sim N\left(p_2, \frac{p_2(1-p_2)}{m}\right)$

$\underbrace{\hat{p}_1 - \hat{p}_2}_{\hat{\theta}} \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{m}\right)$

$1-\alpha = P\left(L < \underbrace{p_1 - p_2}_{\theta} < U\right)$

$U = \hat{p}_1 - \hat{p}_2 + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}}$

$L = \hat{p}_1 - \hat{p}_2$ —— // ——