Probability and Statistics exam demo variant

Single choice or free response, easier problems

1. Suppose that the 95% two-sided confidence interval is constructed for the mean of a normal population with unknown variance using a sample of 9 elements. If the margin of error of this interval is m, what will be the margin of error of the 99% two-sided confidence interval for the population mean constructed using the same sample?

2. A random sample of 10 items is taken from a normal population. The sample had a mean of 82 and a standard deviation is 26. Which is the appropriate 99% confidence interval for the population mean?

   - $82 \pm z_{0.005}(26)$
   - $82 \pm t_{0.005}(26)$
   - $82 \pm z_{0.01}(26/\sqrt{10})$
   - $82 \pm t_{0.005}(26/\sqrt{10})$
   - none of the above

3. What is the probability that the sample mean of a sample of 10 elements obtained from a normal population with mean 0.1 and variance 1 will be negative?

4. Below is the joint distribution of two random variables, X and Y. Find $\mathbb{E}(XY)$.

	$Y = 2$	$Y = 4$
$X = 1$	0.1	0.3
$X = 2$	0.2	0.2
$X = 3$	0.1	0.1

5. Suppose we want to test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 \neq \mu_2$ for the means of two independent normal populations with unknown but equal variances. We apply this test to two samples with $n = m = 5$ elements in each. What will be the rejection region for the value of the statistic at the 5% significance level?

Free response section

Joint distribution discrete free response

Suppose we have a joint PMF of random vector $(X, Y)^\top$ in the following form:

	$Y = 0$	$Y = 2$	$Y = 4$
$X = 0$	0.1	0.1	0
$X = 2$	0.1	0.4	0.1
$X = 4$	0	0.1	0.1

- Define marginal probability distribution for each random variable.
- Check whether variables are independent.
- Define the mean and the variance for each random variable X and Y, the covariance and correlation coefficient for these random variables.
- Find $\mathbb{E}(R)$, where $R = X^2 + Y^2$, by any method you know.
- Find $\mathbb{E}(4X + 2Y)$.

Joint distribution continuous free response

Random variables X and Y have a following joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} c + 0.1xy, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

- Find constant c such that function $f_{X,Y}(x, y)$ is a valid p. d. f.,
- Are X and Y independent?

Confidence interval free response

A manufacturer of light bulbs wants to estimate the mean length of life of a new type of bulb that is designed to be extremely durable. The firm's engineers test nine of these bulbs and find that the mean length of life is $5,200$ hours with a standard deviation of 150 hours. Previous experience indicates that the lengths of life of individual bulbs of a particular type are normally distributed. Construct a 90% confidence interval for the average length of life of all bulbs of this new type.

Hypothesis testing free response

A manufacturer claims that through the use of a fuel additive, automobiles should achieve on average an additional 3 miles per gallon of gas. A random sample of 100 automobiles was used to evaluate this product. The achieved sample mean increase in miles per gallon was 2.4, and the sample standard deviation was 1.8 miles per gallon. Test the null hypothesis that the population mean is at least 3 miles per gallon. Find p-value of this test and interpret your findings. How will your answer change if you are to make a two-tailed test with $H_0 : \mu = 3$?

Regression free response

For a sample of twenty monthly observations, a financial analyst wants to regress the percentage rate of return Y of the common stock of a corporation on the percentage rate of return X of the S. & P. 500 index. The following information is available:

$$\sum_{i=1}^{20} y_i = 22.6 \quad \sum_{i=1}^{20} x_i = 25.4 \quad \sum_{i=1}^{20} x_i^2 = 145.7 \quad \sum_{i=1}^{20} x_i y_i = 150.5 \quad \sum_{i=1}^{20} y_i^2 = 196.2.$$

- Estimate the linear regression of Y on X.
- Interpret the slope of the sample regression line.
- Interpret the intercept of the sample regression line.